New Method of Extracting non-Gaussian Signals in the CMB

Jiun-Huei Proty Wu

Astronomy Department, University of California, Berkeley, CA 94720-3411, USA

Abstract. Searching for and charactering the non-Gaussianity (NG) of a given field has been a vital task in many fields of science, because we expect the consequences of different physical processes to carry different statistical properties. Here we propose a new general method of extracting non-Gaussian features in a given field, and then use simulated cosmic microwave background (CMB) as an example to demonstrate its power. In particular, we show its capability of detecting cosmic strings.

With the cosmological principle as the basic premise, two currently competing theories for the origin of structure in the universe are inflation [1] and topological defects [2,3]. Although the recent CMB observations seem to have favored the former [4], the latter can still coexist with it. In particular, the observational verification of defects will have certain impact to the grand unified theory, since they are an inevitable consequence of the spontaneous symmetry-breaking phase transition in the early universe. In addition to the conventional study of the power spectra of cosmological perturbations, another way to distinguish these models is via the search for intrinsic NG—while the standard inflationary models predict Gaussianity, theories like isocurvature inflation [5] and topological defects [6] generate NG. Here we shall propose a new method of extracting the NG from a given field [7], and then apply it to the CMB, which is arguably the cleanest cosmic signals [8].

The new method aims to nothing but removing the 'Gaussian' components: the mean and the power. Using an n-dimensional field $\Delta(\mathbf{x})$ as an example, the method first Fourier transforms $\Delta(\mathbf{x})$ to yield $\widetilde{\Delta}(\mathbf{k})$. Then the power spectrum can be estimated as $C_k = \langle|\widetilde{\Delta}(\mathbf{k})|^2\rangle_k/V^n$, where V^n is the n-dimensional volume of the field and $k \equiv |\mathbf{k}|$. Next we define and calculate ($\forall\mathbf{k}$ with $C_k \neq 0$)

$$\widetilde{\Delta}_P(\mathbf{k}) = \left[\widetilde{\Delta}(\mathbf{k}) - \widetilde{\Delta}(\mathbf{0})\delta(\mathbf{k})\right] C_k^{-1/2} P_k^{1/2}, \tag{1}$$

where $\delta(\mathbf{k})$ is a Dirac Delta and P_k is a given function of k. Finally, $\widetilde{\Delta}_P(\mathbf{k})$ is transformed back to the real space $\Delta_P(\mathbf{x})$. Now the field Δ_P has a mean $\overline{\Delta}$ equal to zero and a power spectrum renormalized to P_k. For the simplest case $P_k = 1$, the field Δ is 'whitened' in the Fourier space, and we shall use the superscript 'W' to

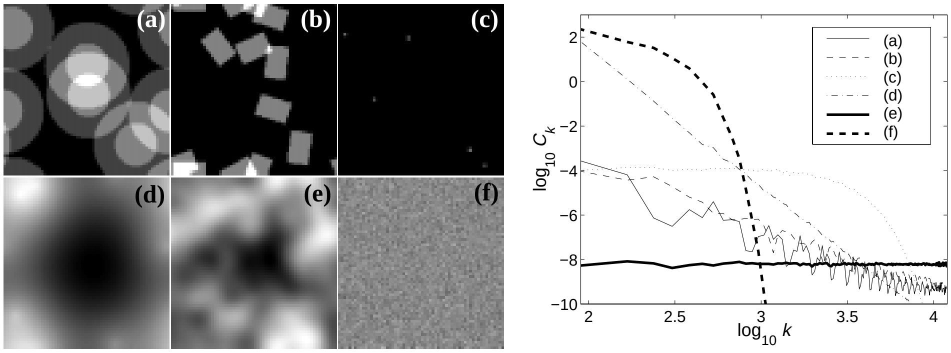

FIGURE 1. Six different components in a simulated CMB map, and their power spectra.

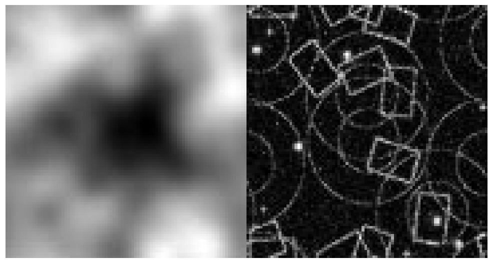

FIGURE 2. A simulated CMB map Δ (left), and the extracted NG signal $(\Delta^{\rm W})^2$ (right).

denote such whitened fields. In the real space, this means $\Delta = \Delta^{\rm W} \otimes D + \overline{\Delta}$, where $\otimes$ denotes a convolution, and $D \equiv \int dk^n C_k^{1/2} e^{i\mathbf{k}\cdot\mathbf{x}}$. Thus Δ is now decomposed into two parts: the 'Gaussian components', $\overline{\Delta}$ and D, which carry the information in the mean and the power spectrum, and the 'NG component', $\Delta^{\rm W}$, which possesses all the remaining information. Therefore, if Δ is a Gaussian field, then all samples in $\Delta^{\rm W}$ should appear uncorrelated as pure white noise. Otherwise $\Delta^{\rm W}$ would contain all the non-Gaussian features [7]. We note that the above treatment can be easily converted to the conventional multipole transform for the CMB, although we shall continue to use the Fourier convention, which is appropriate for small CMB fields. In this case, we have $\ell \equiv k$ and $C_\ell \equiv C_k$. We also notice that the above new method is equivalent to the matrix manipulation $\mathbf{d}_P = \mathbf{P}^{1/2}\mathbf{C}^{-1/2}\mathbf{d}$, where $\mathbf{d} \equiv \Delta$, and $\mathbf{P}$ and $\mathbf{C}$ are the two-point correlation matrices specified by P_k (with $P_0 = 0$) and C_k respectively. This is similar but different from the Wiener filtering.

We now test this formalism using simulations. Figure 1 shows six simulated CMB components: (a)–(d) (where (d) contains 5 diffuse points) are non-Gaussian, while (e) and (f) are Gaussian. They are then linearly summed to yield $\Delta = \sum_i \Delta_{(i)}$, with RMS ratios ((a)–(f)) $1:1:10:500:1000:0.2$ (Figure 2 left). We then apply our method to obtain the extracted NG signal $\Delta^{\rm W}$ (Figure 2 right).

In a second test, we simulate a CMB field of $(2^\circ)^2$ (Figure 3(c)): $\Delta_{\rm s} = [\Delta W_{\rm p}] \otimes W_{\rm o} + \Delta_{\rm noi}$, where $\Delta = \Delta_{\rm bg}$ (Gaussian background) $+\Delta_{\mathcal{S}\rm ISW}$ (string-induced CMB [9]; Figure 3(b)) $+\Delta_{\rm pnt}$ (point source; Figure 3(a)) with RMS ratios $5:1:2$, $\Delta_{\rm noi}$ is a 5% noise, and W_i with $i =$ 'p' and 'o' denote the primary and observing beams respectively. The whitened field $\Delta_{\rm s}^{\rm W}$ is shown in Figure 3 (d).

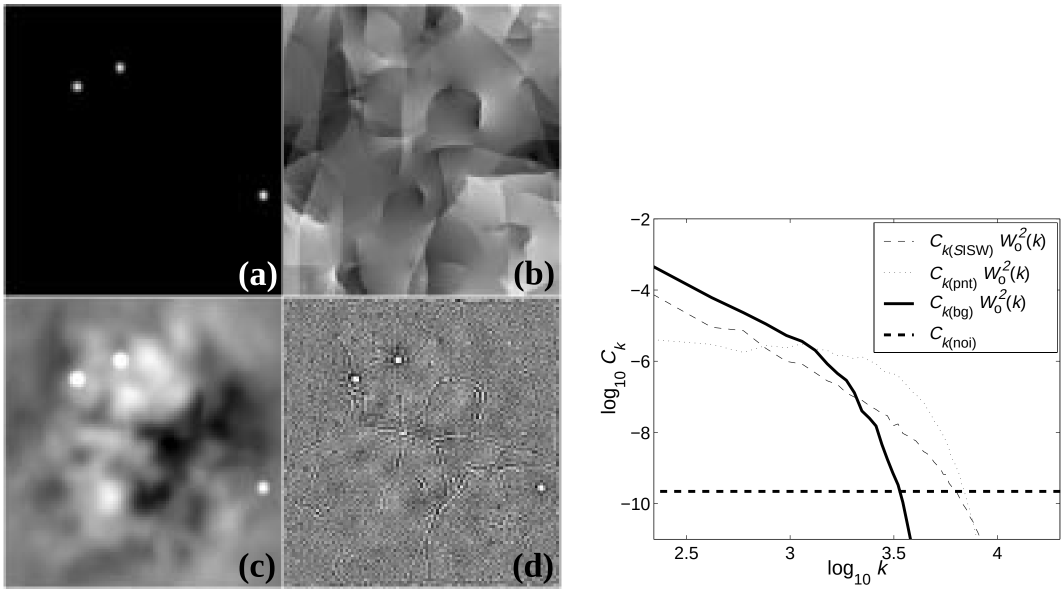

FIGURE 3. Simulated CMB, the extracted NG signal (d), and the power spectra (right).

With even more tests, the main observation remains the same: in a field $\Delta = \Delta_{(\mathrm{G})} + \Delta_{(\mathrm{NG})}$, regardless how stronger the $\Delta_{(\mathrm{G})}$ is, the NG features of $\Delta_{(\mathrm{NG})}$ can always show up in the whitened field Δ^{W} as long as $C_{k(\mathrm{NG})}$ dominates $C_{k(\mathrm{G})}$ within a certain range of k. In fact, this can be analytically proved [7]. In addition, the NG features of uncorrelated NG components do not mix up in the extracted NG field Δ^{W}, even if some of them dominate the others in power.

Finally we notice that according to equation (1), in principle we can design a 'window function' P_k to keep the power only on scales where the NG components of a field dominate. However, in general we do not know what these scales are and thus taking $P_k = 1$ is optimal. This may even enable us to find the NG signals of unknown physical processes. We acknowledge the support from NSF KDI Grant (9872979) and NASA LTSA Grant (NAG5-6552).

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